Planar Porous Resistance Setup Guidelines

This Guide is intended as a supplement to Coolit User's Manual. The purpose of this document is to provide guidelines to obtaining flow resistance information required in specifying Planar Porous Resistance (PPR) components in Coolit.

PPR is a porous media based model, which is used to simulate perforated plates and vents. The main purpose of using PPR is to reduce the number of grid cells required to solve a problem. For completeness, we included the PPR governing equations:

$$\rho C \frac{\mathbf{V}|\mathbf{V}|}{2} = -\nabla p - \frac{\mu}{\kappa}\mathbf{V} \tag{1}$$

In this document, we will show you how to create a wind tunnel in Coolit to obtain the required parameters. We will also provide you with empirical formulas that we have derived for staggered and inline perforated plates.

1. Empirical Formulas for PPR

In this study we considered two types of perforated plates: staggered and inline, Figure 1.

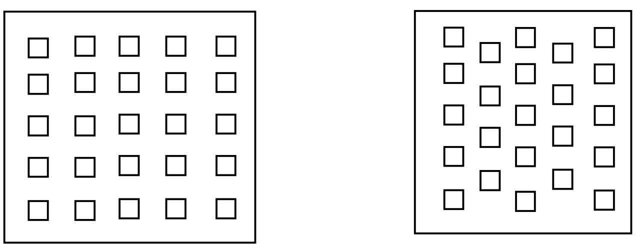

Figure 1. Examples of inline (left) and staggered perforated plates.

To obtain empirical formulas we ran a series of wind-tunnel tests (two typical setups are attached) with fixed percent open, ϕ, defined as the ratio of free and total areas of the perforated plate, and with the varying approach velocity, V. The pressure drop can be obtained from eq. (1) as follows,

$$\Delta p \approx \frac{\mu H}{\kappa}\mathbf{V} + \rho CH \frac{\mathbf{V}|\mathbf{V}|}{2} \tag{2}$$

where Δp is the pressure drop across the PPR, *Pa*, μ is the dynamic viscosity of fluid (air), $N*s/m^2$, H is the thickness of the PPR, *m*, κ is the permeability coefficient, m^2, V is the approach velocity, *m/s*, ρ is the density of fluid (air), kg/m^3, and C is the inertial resistance factor, m^{-1}. Note, that the *CH* term is the standard (dimensionless) loss coefficient. The unknowns in (2) are κ and C, which are required in Coolit's PPR Properties dialog.

To simplify the notation in the following we will use,

$\Delta p = aV + bV^2$, where $a = \mu H/\kappa$ and $b=\rho CH / 2$. (3)

Our aim is to obtain *a* and *b* coefficients in (3) as a function of percent open, ϕ, so that the resulting pressure drop is within 20% of the pressure drop computed by Coolit for Plates with perforations. Once you determine *a* and *b* for your range of velocities and percent open from the formulas below, you can compute κ and C from (3). In the event of $a=0$, κ is set to some large number, e.g. 10^{20}.

1.1 Inline Plate results

	a		*B*	
V = [0.01; 5]	$0.275\phi^2 - 0.696\phi + 0.473$	ϕ = [0.1; 0.9]	$0.5213\phi^{-2.443}$	ϕ = [0.1; 0.5)
			$-43.14\phi^3 + 108.06\phi^2 - 92.31\phi + 27.16$	ϕ = [0.5; 0.9]
V = [0.5; 5]	0	ϕ = [0.1; 0.75]	$0.610\ \phi^{-2.398}$	ϕ =[0.1;0.35]
			$69.38\ \exp(-6.350\phi)$	ϕ =[0.35;0.75]

In this table, the ranges of velocities and percent opens for which the formulas are valid are shown in brackets after respective variables. A square bracket means that the value is included in the range, a parenthesis - that it is not. For example, you have a perforated plate with inline holes, an approach velocity around 1 m/s and percent open 0.5 (i.e. 50%). You have an option of using either the second formula in line 1 or the second formula in line 2:

Choice 1: $a = 0.275\phi^2 - 0.696\phi + 0.473$ and $b = -43.14\phi^3 + 108.06\phi^2 - 92.31\phi + 27.16$.
Choice 2: $a = 0$ and $b = 69.38\ \exp(-6.350\phi)$.

From the table for $\phi = 0.5$, we observe that the error at V=1 m/s for Choice 1 is 3.7%, while for Choice 2 it is 6.6%. We also note that the error for Choice 1 grows rapidly away from V=1, while for Choice 2 it is relatively stable. Thus, if you are confident that the approach velocity is close to 1 m/s, Choice 1 is preferable, while if there is a considerable uncertainty for V, the Choice 2 formula will provide a better prediction.

The following tables present results computed using the above formulas in comparison to results computed by Coolit. The nomenclature is as follows: Δp is the pressure drop computed using Coolit, Δp(1) is the pressure drop computed using the formulas in the first row of the table above, Δp(2) is the pressure drop computed using the formulas in the second row of the table above, and err, % is the percent error between Δp and Δp(1), and between Δp and Δp(2), respectively. Note, that for illustration purposes, we sometimes used the formulas outside of their recommended range of applicability.

$\phi = 0.1$

V	Δp	Δp (1)	err,%	Δp (2)	err,%
0.01	1.79e-02	0.185E-01	3.5	0.153E-01	14.8
0.10	1.27e+00	0.149E+01	17.0	0.153E+01	20.1
0.50	3.42e+01	0.363E+02	6.3	0.381E+02	11.5
1.00	1.39e+02	0.145E+03	4.3	0.153E+03	9.7
3.00	1.28e+03	0.130E+04	1.7	0.137E+04	7.2
5.00	3.64e+03	0.362E+04	0.6	0.381E+04	4.8

$\phi = 0.25$

V	Δp	Δp (1)	err,%	Δp (2)	err,%
0.01	4.97e-03	0.470E-02	5.4	0.169E-02	65.9
0.03	2.15e-02	0.234E-01	8.6	0.153E-01	29.1
0.05	4.90e-02	0.543E-01	10.9	0.424E-01	13.5
0.10	1.70e-01	0.186E+00	9.3	0.169E+00	0.3
0.50	4.32e+00	0.401E+01	7.1	0.424E+01	1.9
1.00	1.79e+01	0.157E+02	12.1	0.169E+02	5.3
3.00	1.69e+02	0.140E+03	17.4	0.153E+03	9.8
5.00	4.91e+02	0.387E+03	21.2	0.424E+03	13.7

$\phi = 0.35$

V	Δp	Δp (1)	err,%	Δp (2)	err,%
0.01	3.40e-03	0.331E-02	2.7	0.752E-03	77.9
0.10	8.40e-02	0.941E-01	12.0	0.752E-01	10.5
0.50	1.74e+00	0.183E+01	4.9	0.188E+01	8.0
1.00	7.11e+00	0.704E+01	1.0	0.752E+01	5.7
3.00	6.86e+01	0.618E+02	10.0	0.676E+02	1.4
5.00	1.94e+02	0.171E+03	12.0	0.188E+03	3.1

$\phi = 0.5$

V	Δp	Δp (1)	err,%	Δp (2)	err,%
0.01	2.10e-03	0.220E-02	4.8	0.290E-03	86.2
0.03	7.30e-03	0.818E-02	12.0	0.261E-02	64.3
0.10	3.90e-02	0.457E-01	17.1	0.290E-01	25.6
0.50	6.60e-01	0.754E+00	14.2	0.725E+00	9.8
1.00	2.72e+00	0.282E+01	3.7	0.290E+01	6.6
3.00	2.80e+01	0.242E+02	13.5	0.261E+02	6.8
5.00	7.98e+01	0.667E+02	16.5	0.725E+02	9.2

$\phi = 0.65$

V	Δp	Δp (1)	err,%	Δp (2)	err,%
0.01	1.50e-03	0.146E-02	2.4	0.112E-03	92.5
0.10	2.20e-02	0.233E-01	6.1	0.112E-01	49.2
0.50	2.90e-01	0.310E+00	6.9	0.280E+00	3.6
1.00	1.07e+00	0.110E+01	3.1	0.112E+01	4.5
3.00	1.00e+01	0.911E+01	8.9	0.101E+02	0.7
5.00	2.94e+01	0.248E+02	15.5	0.280E+02	4.9

$\phi = 0.75$

V	Δp	Δp (1)	err,%	Δp (2)	err,%
0.01	1.10e-03	0.111E-02	0.7	0.593E-04	94.6
0.10	1.50e-02	0.157E-01	4.6	0.593E-02	60.5
0.50	1.70e-01	0.181E+00	6.3	0.148E+00	12.8
1.00	5.90e-01	0.617E+00	4.6	0.593E+00	0.5
3.00	5.26e+00	0.492E+01	6.4	0.534E+01	1.4
5.00	1.55e+01	0.133E+02	14.1	0.148E+02	4.4

$\phi = 0.9$

V	Δp	Δp (1)	err,%	Δp (2)	err,%
0.01	7.24e-04	0.710E-03	2.0	0.229E-04	96.8
0.10	8.84e-03	0.854E-02	3.4	0.229E-02	74.1
0.50	7.69e-02	0.748E-01	2.7	0.572E-01	25.7
1.00	2.26e-01	0.230E+00	1.7	0.229E+00	1.2
3.00	1.57e+00	0.165E+01	5.3	0.206E+01	31.1
5.00	4.33e+00	0.436E+01	0.7	0.572E+01	32.0

1.2 Staggered Plate Results

The nomenclature for staggered plate results is the same as for the inline plates. Note, that the upper limit of percent open for staggered plates is 0.5.

	A		*B*	
V = [0.01; 5]	$0.4777+0.3367\phi-4.22\phi^{2}+4.833\phi^{3}$	$\phi=[0.1;\ 0.5]$	$0.5051\phi^{-2.4507}$	$\phi=[0.1;\ 0.5]$
V = [0.5; 5]	0	$\phi=[0.1;\ 0.5]$	$0.563\ \phi^{-2.430}$	$\phi=[0.1;0.5]$

$\phi = 0.1$

V	Δp	Δp (1)	err,%	Δp (2)	err,%
0.01	1.84e-02	0.190E-01	3.3	0.152E-01	17.6
0.10	1.26e+00	0.147E+01	16.9	0.152E+01	20.3
0.50	3.36e+01	0.359E+02	6.8	0.379E+02	12.7
1.00	1.37e+02	0.143E+03	4.4	0.152E+03	10.6
3.00	1.28e+03	0.128E+04	0.4	0.136E+04	6.5
5.00	3.54e+03	0.357E+04	0.8	0.379E+04	7.0

$\phi = 0.2$

V	Δp	Δp (1)	err,%	Δp (2)	err,%
0.01	6.97e-03	0.676E-02	3.1	0.281E-02	59.7
0.10	2.82e-01	0.302E+00	7.2	0.281E+00	0.3
0.50	6.91e+00	0.673E+01	2.6	0.703E+01	1.7
1.00	2.87e+01	0.265E+02	7.7	0.281E+02	2.0
3.00	2.66e+02	0.236E+03	11.3	0.253E+03	4.9
5.00	7.50e+02	0.654E+03	12.8	0.703E+03	6.3

$\phi = 0.3$

V	Δp	Δp (1)	err,%	Δp (2)	err,%
0.01	4.40e-03	0.426E-02	3.2	0.105E-02	76.1
0.10	1.20e-01	0.129E+00	7.9	0.105E+00	12.5
0.50	2.77e+00	0.258E+01	6.9	0.262E+01	5.3
1.00	1.16e+01	0.999E+01	13.9	0.105E+02	9.5
3.00	1.09e+02	0.879E+02	19.4	0.945E+02	13.3
5.00	3.03e+02	0.243E+03	19.8	0.262E+03	13.4

$\phi = 0.4$

V	Δp	Δp (1)	err,%	Δp (2)	err,%
0.01	2.96e-03	0.294E-02	0.6	0.522E-03	82.4
0.10	6.31e-02	0.724E-01	14.7	0.522E-01	17.3
0.50	1.17e+00	0.132E+01	12.5	0.130E+01	11.5
1.00	4.71e+00	0.502E+01	6.5	0.522E+01	10.8
3.00	5.11e+01	0.437E+02	14.5	0.470E+02	8.1
5.00	1.45e+02	0.121E+03	16.9	0.130E+03	10.0

$\phi = 0.5$

V	Δp	Δp (1)	err,%	Δp (2)	err,%
0.01	2.20e-03	0.223E-02	1.3	0.303E-03	86.2
0.10	4.10e-02	0.471E-01	15.0	0.303E-01	26.0
0.50	6.50e-01	0.788E+00	21.2	0.758E+00	16.7
1.00	2.52e+00	0.296E+01	17.3	0.303E+01	20.4
3.00	2.49e+01	0.254E+02	2.2	0.273E+02	9.7
5.00	7.31e+01	0.700E+02	4.2	0.758E+02	3.8

2. Resistance Coefficients from Wind Tunnel Experiment

2.1 Wind Tunnel Construction

If you would prefer to obtain resistance coefficients from a numerical experiment rather than relying on the empirical formulas above or on those from handbooks, you will need to build two wind tunnels in Coolit: one for PPR and the other for perforated plate. In both cases, a wind tunnel is built as a rectangular tube made of Symmetry sides with an inlet (Fan) and outlet (Open). The PPR wind tunnel can be relatively short with coarse grids: the accuracy of predicted pressure drop in fact does not depend on either tunnel length or grid density. It is controlled exclusively by κ and C resistance coefficients that you specify in Coolit and the entire pressure drop occurs over the PPR.

The wind tunnel for perforated plate has to be built somewhat differently. The pressure drop in real perforated plates occurs up- and downstream of the plate as the flow is stopped before complete blockages, being forced into narrow holes and then expanding out of the holes forming vortices downstream. To correctly predict the pressure drop over the perforated plate, the flow up- and downstream of the plate has to be resolved. Therefore, you will need a longer wind tunnel with a finer grid. The length of the wind tunnel is determined by the flow pattern downstream of the perforated plate. It is desirable that the tunnel be long enough to prevent inflow through the outlet section. Since it is impossible to determine the tunnel length a priori, you may need to run several experiments with different tunnel lengths for the maximum anticipated approach velocity.

In the perforated plate case, you will also need to make a model of the plate. Normally, perforations are made with a symmetry pattern, examples of which are shown in Figure 1 above. If that's the case, you can save time by modeling only a symmetry element of the pattern and using symmetry walls for the wind tunnel. Examples of symmetry elements for Figure 1 are shown in Figure 2. If a perforated plate lacks a symmetry pattern, you can take a representative (sufficiently large) piece of the plate in order to determine the pressure drop.

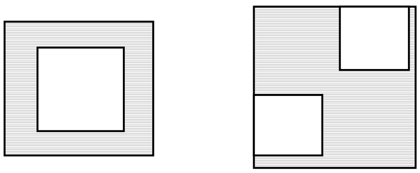

Figure 2. Examples of symmetry elements for inline and staggered perforated plates in Figure 1.

The pressure difference required by the experiments is determined in both cases as the difference between the average pressure over the outlet minus the average pressure over the inlet.

2.2 Calculation of Resistance Coefficients

Once you have established the minimum length of the wind tunnel, you should run several computer experiments, using different approach velocities within the anticipated range in order to determine the resistance coefficients in eq. (3). If the range of anticipated velocities is relatively small and it fits in either linear or quadratic portion of the pressure curve, you can save time by assuming that one of the coefficients in (3) equals zero.

Typically, for *device* velocities (device velocity is equal to the approach velocity divided by percent open) less than 0.5 m/s the pressure is a linear function of velocity and you can assume the inertial resistance factor C to be zero. Above 0.5 m/s, the resistance is well described by the quadratic term alone, hence in eq. (3) $a = \mu H/\kappa = 0$, i.e. the permeability is set to some very large number.

Now we will consider all of the above cases individually.

2.2.1 Low Velocity Case

In low velocity cases, the viscous (linear) forces are dominant. For example, assume a natural convection environment and the range of expected device velocities in the 0.01 -- 0.2 m/s range. You can safely assume the pressure drop to be a linear function of velocity, i.e. b = C = 0 and

$\Delta p = aV$, where $a = \mu H/\kappa$.

Only one data point is required to determine a. However, as in any experiment, whether a physical or numerical, it is better to obtain several data points and then average the result. Let us assume that you ran 3 experiments and computed three Δp's for 3 V's, that is

$$\Delta p_1 = aV_1$$
$$\Delta p_2 = aV_2$$
$$\Delta p_3 = aV_3.$$

Before computing the average, you should normalize the above equations to ensure that their contributions have a comparable weight. Dividing each equation by respective velocities is one way of doing this, e.g. $\Delta p_1 / V_1 = a$, etc. The average a is then simply

$a = (\Delta p_1 / V_1 + \Delta p_2 / V_2 + \Delta p_3 / V_3) / 3$.

The general case of N-experiments would result in: $a = \dfrac{1}{N}\sum_{i=1}^{N}\dfrac{\Delta p_i}{V_i}$. Finally, you can determine the permeability coefficient from: $\kappa = \mu H / a$.

2.2.2 High Velocity Case

In high velocity cases, inertial (quadratic) forces dominate. For example, imagine a forced convection problem with the expected range of device velocities from 1 m/s to 3 m/s. We assume that the viscous forces are negligible with $a = 0$ and hence $\kappa = \infty$. In practice, κ is set to some large number, e.g. 10^{20}. Following the procedure described above, and assuming N-experiments, we get

$b = \dfrac{1}{N}\sum_{i=1}^{N}\dfrac{\Delta p_i}{V^2{}_i}$, and $C = 2b / (\rho H)$.

2.2.3 General Case

Finally, we will consider the most general case in which the velocity range covers both linear and non-linear regions and, hence, neither coefficient in eq. (3) can be assumed to be zero. Applying the least squares method to find the a and b coefficients, we will minimize

$$\sum_{i=1}^{N}\left[\Delta p_i / V_i^2 - a / V_i \ - b\right]^2 = \min$$

where N is the number of numerical experiments, N>2. The resulting system would be:

$$\sum_{i=1}^{N}\left[\Delta p_i / V_i^2 - a / V_i \ - b\right] = 0 \tag{4}$$

$$\sum_{i=1}^{N}\left[\Delta p_i / V_i^2 - a / V_i \ - b\right] / V_i \ = 0 \tag{5}$$

The solution of system (4)-(5) is

$$a = \frac{\begin{vmatrix} \sum \frac{\Delta p_i}{V_i^2} & N \\ \sum \frac{\Delta p_i}{V_i^3} & \sum \frac{1}{V_i} \end{vmatrix}}{\begin{vmatrix} \sum \frac{1}{V_i} & N \\ \sum \frac{1}{V_i^2} & \sum \frac{1}{V_i} \end{vmatrix}}, \quad b = \frac{\begin{vmatrix} \sum \frac{1}{V_i} & \sum \frac{\Delta p_i}{V_i^2} \\ \sum \frac{1}{V_i^2} & \sum \frac{\Delta p_i}{V_i^3} \end{vmatrix}}{\begin{vmatrix} \sum \frac{1}{V_i} & N \\ \sum \frac{1}{V_i^2} & \sum \frac{1}{V_i} \end{vmatrix}}.$$

The permeability coefficient and inertial resistance factor can be found from: $\kappa = \mu H / a$ and $C = 2b / (\rho H)$.